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# Optimal and Heuristic Algorithms of Planning of Low-rise Residential Buildings

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**Abstract.** The problem of the optimal layout of low-rise residential building is considered. Each apartment must be no less than the corresponding apartment from the proposed list. Also all requests must be made and excess of the total square over of the total square of apartment from the list must be minimized. The difference in the squares formed due to with the discreteness of distances between bearing walls and a number of other technological limitations. It shown, that this problem is NP-hard. The authors built a linear-integer model and conducted her qualitative analysis. As well, authors developed a heuristic algorithm for the solution tasks of a high dimension. The computational experiment was conducted which confirming the efficiency of the proposed approach. Practical recommendations on the use the proposed algorithms are given.

## INTRODUCTION

The considered problem belong to number of problem of a discrete optimization [1] and significantly related problem of the linear optimal cutting of industrial sheet material [2].

Now in Russia, it implemented the government program of resettlement of dwellers from emergency and dilapidated buildings. The government program finances the same count of meters, which had displaced dweller. The construction company will pay additional meters, providing dweller new apartment in new house instead of old apartment.

The resettled dwellers have the residential squares of different sizes. Due to naturally technically restrictions, all apartments in one building to make the different are impossible. Therefore, before designers set the task of the planning apartment optimization in order to additional squares were minimal. The residential buildings of section type are considered. We assume that the residential apartments have a squared shape and are located in section at four. In each section has the entryway. Also we assume that all residential apartments has same width, but has different lengths. In the part of section where no entryway will be located two-bedroom apartments and in the opposite one-bedroom apartments (Figure 1). The square of the entryway is fixed and same for all sections in building. One-bedroom and two-bedroom apartments to be arranged thus that were performed all requests for required apartments and the total length of sections was minimal. The similar problems arise when designing and cutting of the ribbon, in the planning of loading vehicles and so on.

## MATHEMATICAL FORMULATION OF THE PROBLEM

There are  $n$  sections of fixed width  $2c$  and valuable of length  $l_i$ ,  $i \in \overline{1, n}$ . For completing  $n$  sections required  $4n$  rectangles with width  $c$ , which it allocated four in each section (Figure 1). We should to define two rectangles in upper part and two rectangles in lower part. In addition, in each section in lower part contains additional one rectangle of constant size  $c \times g$  (entryway). In the task also there are the additional restriction: the difference of lengths of the

appropriate rectangles in sections should not exceed value  $d$ . This restriction is justified by the fact that the necessary space for the installation of entrance doors in the apartments of the upper part of the section. Let the lengths of rectangles, located in upper part of section, are denoted by  $\hat{b}_{i,1}$ ,  $\hat{b}_{i,2}$ , lengths of rectangles, located in upper part of section, are denoted by  $\hat{a}_{i,1}$ ,  $\hat{a}_{i,2}$ . The thickness of wall between rectangles is denoted by  $q$ .

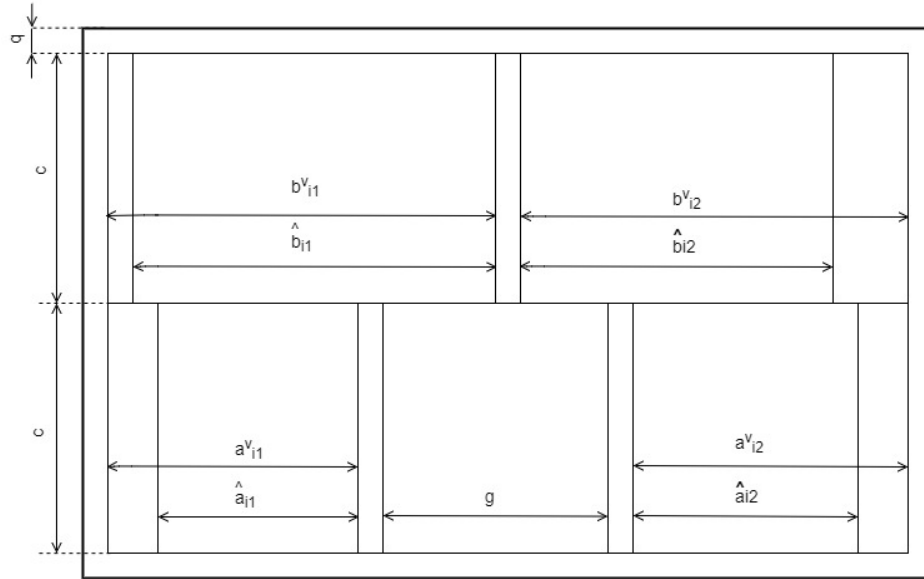


FIGURE 1.

We assume that the following conditions must be satisfied:

$$\begin{cases} \hat{a}_{i,1} + \hat{a}_{i,2} + g + 3q \leq l_i, \\ \hat{b}_{i,1} + \hat{b}_{i,2} + 2q \leq l_i, \\ i \in \overline{1, n}. \end{cases} \quad (1)$$

We introduce the definition of modified lengths

$$\begin{cases} b_{i,1}^v \geq \hat{b}_{i,1}, \\ b_{i,2}^v \geq \hat{b}_{i,2}, \\ a_{i,1}^v \geq \hat{a}_{i,1}, \\ a_{i,2}^v \geq \hat{a}_{i,2} \end{cases}$$

with restrictions

$$\begin{cases} b_{i,1}^v - a_{i,1}^v \geq d, \\ b_{i,2}^v - a_{i,2}^v \geq d, \\ i \in \overline{1, n}. \end{cases} \quad (2)$$

We rewrite conditions (1) taking into account restrictions (2):

$$\begin{cases} a_{i,1}^v + a_{i,2}^v + g + 3q = l_i, \\ b_{i,1}^v + b_{i,2}^v + 2q = l_i, \\ i \in \overline{1, n}. \end{cases} \quad (3)$$

Equality upper and lower blocks gives the equation:

$$a_{i,1}^v + a_{i,2}^v + g + 3q = b_{i,1}^v + b_{i,2}^v + 2q. \quad (4)$$

The added lengths will be equal to

$$L_i = b_{i,1}^v + b_{i,2}^v + a_{i,1}^v + a_{i,2}^v - \hat{b}_{i,1} + \hat{b}_{i,2} - \hat{a}_{i,1} + \hat{a}_{i,2}.$$

Our task is to minimize the following quantity

$$\sum_{i=1}^n L_i \longrightarrow \min. \quad (5)$$

We construct the integer linear programming problem for the solution of problem (5).

We introduce the binary values  $x_{ki}^{b_1}, x_{ki}^{b_2}, x_{ki}^{a_1}, x_{ki}^{a_2} \in \{0, 1\}$  in accordance rectangles  $b_1, b_2, a_1, a_2$ , where  $k \in \overline{1, n}$ ,  $i \in \overline{1, m}$ ,  $n$  is the count of sections and  $m$  is the count of rectangles. Thus  $x_{ki}^{b_1} = 1$  signifies that in  $k$  section in place rectangle  $b_1$  is located  $i$  rectangle from the given set of rectangles  $b_i$  and  $x_{ki}^{a_2} = 1$  signifies that in  $k$  section in place rectangle  $a_2$ .

We have same restrictions for each section  $k \in \overline{1, n}$ .

The each rectangle should be use exactly once:

$$\begin{cases} \sum_{i=1}^m x_{ki}^{b_1} = 1, \\ \sum_{i=1}^m x_{ki}^{b_2} = 1, \\ \sum_{i=1}^m x_{ki}^{a_1} = 1, \\ \sum_{i=1}^m x_{ki}^{a_2} = 1. \end{cases}$$

The length of four rectangles  $l$  calculated in the following way:

$$\begin{cases} l_{ki}^{b_1} = \sum_{i=1}^m b_i x_{ki}^{b_1}, \\ l_{ki}^{b_2} = \sum_{i=1}^m b_i x_{ki}^{b_2}, \\ l_{ki}^{a_1} = \sum_{i=1}^m a_i x_{ki}^{a_1}, \\ l_{ki}^{a_2} = \sum_{i=1}^m a_i x_{ki}^{a_2}. \end{cases}$$

We introduce the definition of modified lengths

$$\begin{cases} l_{vk}^{b_1} \geq l_k^{b_1}, \\ l_{vk}^{b_2} \geq l_k^{b_2}, \\ l_{vk}^{a_1} \geq l_k^{a_1}, \\ l_{vk}^{a_2} \geq l_k^{a_2}. \end{cases} \quad (6)$$

We rewrite restrictions (2) taking into account the definition (6):

$$\begin{cases} l_{vk}^{b_1} - l_{vk}^{a_1} \geq d, \\ l_{vk}^{b_2} - l_{vk}^{a_2} \geq d. \end{cases} \quad (7)$$

Equality upper and lower blocks similarly (4) will be the next:

$$l_{vk}^{b_1} + l_{vk}^{b_2} + 2q = l_{vk}^{a_1} + l_{vk}^{a_2} + g + 3q. \quad (8)$$

Calculation of the added lengths is

$$\bar{L}_k = l_{vk}^{a_1} - l_k^{a_1} + l_{vk}^{a_2} - l_k^{a_2} + l_{vk}^{b_1} - l_k^{b_1} + l_{vk}^{b_2} - l_k^{b_2}.$$

Restrictions for all sections is define what each rectangle should be use exactly in one section:

$$\begin{cases} \sum_{k=1}^n (x_{ki}^{b_1} + x_{ki}^{b_2}) \leq 1, \\ \sum_{k=1}^n (x_{ki}^{a_1} + x_{ki}^{a_2}) \leq 1, \\ i \in \overline{1, m}. \end{cases} \quad (9)$$

The objective function is the following

$$\sum_{k=1}^n \bar{L}_k \longrightarrow \min. \quad (10)$$

In the book [3] the problems of the application of linear programming in the construction sector are considered, but the similar solutions of the considered problems were not found.

## HEURISTIC ALGORITHM

The mathematical model, which reduced to the linear integer programming, effective, when the number of the sections less than twenty. It is advisable to use the approximately algorithms for the number of the sections more than twenty. In this paper, it proposed some the greedy algorithm based on reaching of local optimums.

Initially set the count of iterations of the calculation task in our algorithm.

**Step 1.** We introduce an additional designations: the count of rectangles that are available for lower part denote by  $k$ ; the count of rectangles that are available for upper part denote by  $z$ ; rectangle from the lower part denote by  $a_i, i \in \overline{1, k}$ ; rectangle from the upper part denote by  $b_i, i \in \overline{1, z}$ . If  $z = k$  and  $z$  - even number, then as a result our computation will be the optimal variant of planning  $z/2$  sections. If  $z = k$  and  $z$  - odd number, then as a result our computation will be the optimal variant of planning  $(z - 1)/2$  sections. If  $z \neq k$ , then as a result our computation will be the optimal variant of planning  $\min(z/2, k/2)$  sections. The practical results shown that selection method the first rectangle is crucial. The strict choose of the medium valuable not allow us to avoid encounter into the local minimum region from which not escape on the following iterations. The practical investigations gave good results in case of choice the first rectangle: first, medium and last values. In modified version of the algorithm, we have learned to compare received results and to choose optimal of them. We are looking for the remaining three rectangles from condition:  $l_i \rightarrow \min$  and take into account the additional restrictions. As a result, the first section will be form.

**Step 2.** We fill the following sections from remaining rectangles similarly to step 1.

**Step 3.** As a result of this iterative procedure, we fill the all sections. We remember the received result. On the following iteration we will to compare a current result with the remembered and to choose the best result of them and to remember it.

**Step 4.** Further, we find the maximal summand in function (10). We choice the rectangle from appropriate section thanks to which this summand turned out the largest. Further, we return to the first step with the found rectangle. The computational procedure finishes after the specified number of iterations.

## PLANNING OF LOW-RISE BUILDINGS

The algorithm of planning of low-rise buildings based on our heuristic algorithm the mentioned above by bringing a general task to more specified and already solved. There are  $2n \times m$  rectangles from the upper part and  $2n \times m$  rectangles from the lower part. Each set of rectangles sorted in ascending order. Further, the set of rectangles is divided sequentially on  $m$  groups, when  $m$  - the number of levels of packaging. We choose the representative from each group. The representative is the largest rectangle. The remaining rectangles in group bring to this value, computing at the same additional value. Thus, the task of low-rise planning reduces to known single-floor due to same planning on floors. Also solved task of partition of the set of rectangles on groups when rectangles more then  $2n \times m$ .

For example, there are 17 rectangles  $\hat{a}_i$  and 19 rectangles  $\hat{b}_i$ , of which it is necessary to design two floors. In this example, the optimal planning will contain of  $2 \times 4 \times 2 = 16$  rectangles. Before grouping groups of rectangles by levels it is necessary to define the exceed variants of rectangles, which will be discarded. In our algorithm for any set in case the exceed variants, task of grouping by levels solves as follows: if the exceed variants one or two then it runs comprehensive search of all possible variants of  $m$ -groups with calculation their total additive. At the end of the comprehensive search is selected the variant of grouping in which the additive was minimal. If the exceed variants more than two, then in the set of rectangles  $\hat{a}_i$  are discarded the least and in the set of rectangles  $\hat{b}_i$  are discarded the largest. In both cases is discarded the exceed variants until the exceed variants remains less the three. After this starts comprehensive search mentioned above. After the grouping, we received the next output data: sets of rectangles for the packaging of one floor, estimated additive reduction of values in the group, the lists of the values of rectangles, not included in the optimal set.

## COMPUTATIONAL EXPERIMENT

In the computational experiment were calculated four examples.

**Example 1.** Required to complete three sections of the fourteen rectangles with minimal addition. Is this example there is one exceed rectangle for both parts of sections. Numeric data:

Rectangles  $\hat{a}_i$  : 3.3; 3.9; 4.8; 6.0; 7.2; 8.1; 6.6.

Rectangles  $\hat{b}_i$  : 4.2; 6.6; 8.1; 9.0; 9.9; 11.1; 11.4.

The optimal value of objective function, obtained by linear integer programming for six sections equals 5.1.

The optimal packaging of sections has the form i)  $a_{i,1}^v; a_{i,2}^v; b_{i,1}^v; b_{i,2}^v$  :

1) 3.3; 4.8; 4.8; 6.6; 2) 6.0; 6.6; 8.1; 9.0; 3) 7.2; 8.1; 9.9; 11.1.

Rectangle  $\hat{a}_i$  not included in the optimal set: 3.9.

Rectangle  $\hat{b}_i$  not included in the optimal set: 11.4.

The time on the search by heuristic algorithm amount to 0.004 seconds, by integer linear programming method amount to 0.17 seconds. The value of objective function (7) obtained after the first iteration of heuristic algorithm matches with the optimal result from the linear integer programming (10).

**Example 2.** Required to complete ten sections of the forty rectangles with minimal addition. The optimal value of objective function obtained by heuristic algorithm equals 13.2. The time on the search by heuristic algorithm amount to 0.113 seconds. The optimal value of objective function obtained by integer linear programming method equals 13.2. The time on the search by integer linear programming method amount to 0.27 seconds.

**Example 3.** Required to complete thirty sections of the one hundred twenty rectangles. The time on the search by heuristic algorithm amount to 9.211 seconds. The optimal result was recorded on the second iteration.

**Example 4.** The three-floor planning. Required to complete three floors of the twenty-four rectangles. Twelve rectangles of every type sorted and divided on three groups, which corresponds to the number of floors. The resulting list of representatives of groups is input data for the heuristic algorithm. The time on the search by heuristic algorithm amount to 0.019 seconds.

The mathematical model was implemented in Python 2.7 using the ILOG IBM CPLEX 12.6 library. Calculations were made on the computer with the Intel i7-4771 processor, 32GB memory.

## CONCLUSIONS

In this paper, the authors proposed the mathematical model for the problem of planning of low-rise residential buildings, which is a problem of linear integer programming, and created the appropriate software. For the tasks of high dimension, when the task cannot be solved using the linear integer programming, the heuristic algorithm was developed. The authors also compared results of the exact and heuristic algorithm on tasks of non-high dimension and computational time between heuristic algorithm and integer linear programming method. Four different examples are considered. The illustrated examples shown the efficiency of proposed algorithms for the problem of planning of the low-rise residential building.

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